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Bounded Speed of Propagation for Solutions to Radiative Transfer Equations

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Abstract. The Radiative Transfer Equation is the nonlinear transport equation

$$\partial_t f + \frac{1}{\varepsilon} v \cdot \nabla_x f + \frac{1}{\varepsilon^2}\sigma(\tilde{f})(f - \tilde{f}) = 0, \qquad \text{(RTE)}$$

where $\tilde{f}(x,t) = ⨍ f(x,v,t)dv$ denotes the average of $f(x,.,t)$ on the unit sphere: $|v| = 1$. It describes the absorption and emission of photons in a hot medium. As the mean free path ε goes to 0, f_ε converges to a solution of the Porous Medium Equation $\partial_t u = \Delta F(u)$, with $F'(u) = (N\sigma(u))^{-1}$. Since σ blows up at $u = 0$, solutions to the PME propagate with finite speed. Specifically if $u(\cdot, 0)$ has compact support in $\mathbf{R}^N$ so does $u(\cdot, t)$ for every $t > 0$ and the sets $\Omega(t) = \{x : u(x,t) > 0\}_{t>0}$ form an expanding family as t increases, and $U\Omega(t) = \mathbf{R}^n$. We show in this paper that these propagation properties hold for the solutions f_ε of the RTE for all small ε. Moreover, the growth of the support of f_ε is uniform in ε. Our proofs rely on the construction of explicit solutions (of the travelling wave type) and subsolutions to the RTE. To our knowledge, this is the first example of a kinetic equation with high velocities where localized data propagate always with bounded speed. For Vlasov–Poisson equations, this arises only for particular initial data.

0. Introduction

In this paper we show that the property of finite propagation of disturbances from 0, well known for the solutions of the Porous Medium Equation: $u_t = \Delta u^m$, $m > 1$, holds also for the Radiative Transfer Equation. In order to be specific, we consider

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