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Anomalies in the One-Dimensional Anderson Model at Weak Disorder

Massimo Campanino* and Abel Klein**
Department of Mathematics, University of California, Irvine, Irvine, CA 92717, USA

Abstract. We show that at the special energies $E = 2\cos \pi p/q$, the invariant measure, the Lyapunov exponent, and the density of states can be extended to zero disorder as C^∞ functions in the disorder parameter. In particular, we obtain asymptotic series in the disorder parameter. This gives a rigorous proof of the existence of the anomalies originally discovered by Kappus and Wegner and studied by Derrida and Gardner and by Bovier and Klein.

1. Introduction

The one-dimensional Anderson model is given by the random Hamiltonian

$$H = H_0 + \lambda V \quad \text{on} \quad l^2(\mathbf{Z}),$$

where

$$(H_0 u)(n) = u(n+1) + u(n-1), \quad u \in \mathbf{Z},$$

and the $V(x)$, $x \in \mathbf{Z}$, are independent identically distributed random variables with common probability distribution μ. We will write h for the characteristic function of μ, i.e. $h(t) = \int e^{-itv} d\mu(v)$. The real parameter λ will be called the *disorder*.

In this article we will always assume that μ has finite moments of all orders and is not concentrated in a single point; we normalize μ by

$$\int v d\mu(v) = 0, \quad \int v^2 d\mu(v) = 1.$$

The eigenvalue equation associated with H is

$$u(n+1) + u(n-1) = (E - \lambda V(n))u(n). \tag{1.1}$$

Setting

$$Z(n) = \frac{u(n)}{u(n-1)} \in \dot{\mathbf{R}} = \mathbf{R} \cup \{\infty\},$$

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* Permanent address: Dipartimento di Matematica, Universita de Bologna, p.zz.a S. Donato, 5, I-40126 Bologna, Italy

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