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Quantum Deformation of Lorentz Group

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Abstract. A one parameter quantum deformation $S_\mu L(2,\mathbb{C})$ of $SL(2,\mathbb{C})$ is introduced and investigated. An analog of the Iwasawa decomposition is proved. The compact part of this decomposition coincides with $S_\mu U(2)$, whereas the solvable part is identified as a Pontryagin dual of $S_\mu U(2)$. It shows that $S_\mu L(2,\mathbb{C})$ is the result of the dual version of Drinfeld's double group construction applied to $S_\mu U(2)$. The same construction applied to any compact quantum group G_c is discussed in detail. In particular the explicit formulae for the Haar measures on the Pontryagin dual G_d of G_c and on the double group G are given. We show that there exists remarkable $1-1$ correspondence between representations of G and bicovariant bimodules ("tensor bundles") over G_c. The theory of smooth representations of $S_\mu L(2,\mathbb{C})$ is the same as that of $SL(2,\mathbb{C})$ (Clebsh-Gordon coefficients are however modified). The corresponding "tame" bicovariant bimodules on $S_\mu U(2)$ are classified. An application to $4D_+$ differential calculus is presented. The nonsmooth case is also discussed.

0. Introduction

Despite 60 years of intensive efforts of many eminent physicists quantum theory is not yet fully compatible with the (special) theory of relativity. The failure of the program of constructive quantum field theory is one of many manifestations of this incompatibility. More detailed analysis shows that the difficulty lies in small space distances. It seems that the four-dimensional smooth pseudoriemannian manifold is a good model of our space-time only in the macro-scale. The description of the space-time in sub-micro level may require the new tools provided by a noncommutative generalization of differential geometry [2, 9, 12].

It is not clear what the symmetry properties of the space-time in the sub-micro scale are. The idea that the symmetry properties are described by a quantum group [3, 8, 10] is very attractive. By virtue of the correspondence principle the symmetry group should be a deformation of the Poincaré group.

The Poincaré group is the semidirect product of Lorentz and translation groups. Therefore at the first step we should construct a quantum deformation of