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Schrödinger Operator with a Nonlocal Potential whose Absolutely Continuous and Point Spectra Coexist

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Abstract. We consider the Schrödinger-like operator H in which the role of a potential is played by the lattice sum of rank 1 operators $|v_n\rangle\langle v_n|$ multiplied by $g \tan \pi[(\alpha, n) + \omega]$, $g > 0$, $\alpha \in \mathbb{R}^d$, $n \in \mathbb{Z}^d$, $\omega \in [0, 1]$. We show that if the vector α satisfies the Diophantine condition and the Fourier transform support of the functions $v_n(x) = v(x - n)$, $x \in \mathbb{R}^d$, $n \in \mathbb{Z}^d$, is small then the spectrum of H consists of a dense point component coinciding with $\mathbb{R}$ and an absolutely continuous component coinciding with $[\varrho, \infty)$, where ϱ is the radius of the mentioned support. Besides, we find the integrated density of states $N(\lambda)$ (it has a jump at $\lambda = \varrho$) and zero temperature a.c. conductivity $\sigma_\lambda(\nu)$, that also has a jump at $\lambda = \varrho$ and vanishes faster than any power of the external field frequency ν as $\nu \to 0$ and $\lambda \neq \varrho$.

1. Main Results and Discussion

The present work is devoted to the spectral analysis of an operator

$$H = -(2\pi)^{-2}\Delta + Q \tag{1.1}$$

on $L_2(\mathbb{R}^d)$. Here Δ is the Laplace operator and the operator Q (a pseudopotential) has the form

$$Q = \sum_{n \in \mathbb{Z}^d} t_n |v_n\rangle\langle v_n| , \tag{1.2}$$

$$(|v_n\rangle\langle v_n|\varphi) \ (x) = v_n(x) \ (v_n, \varphi) , \qquad v_n(x) = v(x - n) \ ,$$

$$t_n = \tan \pi[(\alpha, n) + \omega] , \qquad \omega \in [0, 1) \tag{1.3}$$

$$\omega \neq \tfrac{1}{2} - (\alpha, n) \,(\mathrm{mod}\, 1) , \qquad n \in \mathbb{Z}^d , \tag{1.4}$$

the vector $\alpha \in \mathbb{R}^d$ satisfying a Diophantine condition

$$|(\alpha, n) - m| \geq C|n|^{-\beta}, m \in \mathbb{Z}, \qquad n \in \mathbb{Z}^d \backslash \{0\} \tag{1.5}$$

with positive constants C and β.