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Schrödinger Dynamics and Physical Folia of Infinite Mean-Field Quantum Systems

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Abstract. This work deals with the general (i.e. non-equilibrium) Schrödinger dynamics of infinite mean-field quantum systems. It is shown how this dynamics is related to the Hamiltonian flow φ_t^Q on the "classical phase-space" $E \subseteq \mathbb{R}^L$ recently defined by Bona [10] to describe the time evolution of classical (macroscopic) observables of the system. These connections allow us to clarify the structure of the set of all physical folia, a notion introduced by Sewell for the dynamical description of infinite systems in cases where this description is representation-dependent. They also yield a result showing that the Heisenberg picture is the more general approach to such descriptions in the sense that there are more representations in which a Heisenberg dynamics can be defined than ones which allow for the definition of a Schrödinger dynamics. Finally, our theory makes it possible to construct many explicit examples of physical folia; in this connection it is shown that there can be overcountably many inequivalent representations with the same macroscopic dynamical structure.

I. Introduction

Recently, great progress has been made concerning the rigorous formulation of the general, i.e. non-equilibrium dynamics of infinite quantum (lattice) systems with a mean-field interaction.

Such models were first considered by Hepp and Lieb [1], who showed that they can be successfully applied to problems in superconductivity and laser-physics. One interesting feature of infinite mean-field systems is that, due to the extremely long range of the interaction, the Heisenberg dynamics cannot be defined as a *-automorphism group of the C^*-algebra $\mathscr{A}$ of quasilocal observables [2]; or, expressed in the Schrödinger picture, that the "set of physical states" [3] is not all of $S(\mathscr{A})$, the state space of $\mathscr{A}$. While the dynamical description of the system in thermal equilibrium—i.e. in the GNS-representation of KMS- or limiting Gibbs-states—was discussed extensively in the literature [4], the concepts necessary to deal with the general case developed more slowly.

Sewell [3] worked in the Schrödinger picture and used the notion of a "folium"