The Supersymmetry Index and the Construction of Modular Invariants

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Abstract. Relationships between the modular properties of affine G characters and the modular properties of the affine characters of regular subgroups of G are derived by considering the branching functions that appear in calculation of the index of the Dirac-Ramond operator on super-coset models. Various applications of these relationships are described, and in particular a simple algorithm is given for generating modular invariant combinations of characters of affine G at any level by using the shift vector method on suitably chosen Lorentzian, self-dual lattices.

1. Introduction

The construction of modular invariants for arbitrary unitary representations of Kac-Moody algebras has been of interest for some time. Moreover, with the advent of $N = 2$ supersymmetric coset models [1] the construction of such invariants has become of some significance for string theory. My intention here is to introduce a technique that helps one to classify these invariants, and as a by-product yields an elementary computational algorithm for generating vast numbers of (and possibly all) modular invariant combinations of characters for Kac-Moody representations of any level.

The basic idea is decribed in Sect. 2 of this paper and rests on a simple, but powerful identity that can be derived for any supersymmetric coset model. This identity was essentially derived in [2], however the emphasis there was on the properties of the coset model, whereas here the coset model in literally orthogonal to the discussion. This identity will be used to study the modular invariants of affine G-representations via embeddings of regular subgroups, H, of G.

In Sect. 3 a number of applications of the results of Sect. 2 will be given. In particular, one can obtain an elementary derivation of the formula for modular

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