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Semi-Global Solutions of Einstein Equations, Minkowskian Near Past Infinity

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Abstract. On a universe homeomorphic to $V^T =]-\infty, T[x\mathbb{R}^3$, we prove the existence of solutions of Einstein equations, minkowskian near past infinity, if the sources are small enough for some norms. We prove that some of these solutions verify at least the positivity condition ("Weak energy condition") on some domains homeomorphic to V^T.

Introduction

In this paper we prove the existence of solutions of Einstein equations with sources in a universe homeomorphic to $V^T =]-\infty, T[xR^3$, such that hypersurfaces $\{t\}xR^3$ are spacelike and curves $]-\infty, T[x\{x\}$ are timelike, without any hypothesis of symmetry. We assume only the hypothesis of sources tending to 0, if $t \to -\infty$, and "small" enough, for some norms, we shall define.

Such solutions represent universes with creation of matter and the question arises, whether they may possess physical meaning, particularly whether they may verify the positivity conditions. Concerning the condition $T^{\alpha\beta}X_\alpha X_\beta \geqq 0$ for any timelike vector field X_α, where $T^{\alpha\beta}$ is the stress-energy tensor, that is the "weak energy condition" of Hawking and Ellis [11], we prove the existence of a solution, which verifies this positivity condition.

However, if the natural frame of $V^T =]-\infty, T[x\mathbb{R}^3$ is minkowskian as $t \to -\infty$, that is $g_{\alpha\beta}(t,x) \to \eta_{\alpha\beta}$, as $t \to -\infty$, where $\eta_{\alpha\beta}$ is the minkowskian metric, it is dubious that we can obtain solutions verifying the positivity condition on the whole of V^T.

We prove only the existence of solutions verifying the positivity condition on domains D, which, in a natural frame minkowskian as $t \to -\infty$, are defined by:

$$D = \{(t,x) \in R^4 \,|\, t < T \text{ and } |x - x_0| < A + \gamma(T-t)\}, \gamma > 1\}$$

or in a domain D which is a union of such domains, spatially bounded for each value of t. But considering their topology and causal structure, such domains are isomorphic to a V^T.