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On the Density of the Ward Ansätze in the Space of Anti-Self-Dual Yang Mills Solutions

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Abstract. A general patching matrix P for the twistor construction of anti-self-dual Yang–Mills fields is approximated by a terminating Laurent series. The approximate patching matrix $P^{(m)}$ is triangularized (so that it becomes one of the Ward ansätze) and the associated Riemann–Hilbert problem is solved, thereby generating an anti-self-dual solution of the Yang–Mills equations. The approximate patching matrices and the associated (exact) anti-self-dual Yang–Mills solutions are then shown to converge on P and its corresponding solution so that the Ward ansätze forms a dense subset in the solution space in the Weierstrass sense.

I. Introduction

For some time now there has been great interest in anti-self-dual (or self-dual) solutions of the Yang–Mills equations [1–6]. One of the reasons for this interest is that many of the familiar non-linear equations of mathematical physics (for example, the Bogomolny equation, the sine- and sinh-Gordan equations, the Toda-lattice equation, the stationary axial symmetric Einstein and Einstein-Maxwell equations, the KdV equation, the non-linear Schrödinger equation, and others) are symmetry reductions of the anti-self-dual Yang–Mills (ASDYM) equations for various gauge groups [7–10]. The ASDYM equations fall into the class of differential equations which are usually described as integrable or solvable [8]. Many solution generating techniques have been developed for the ASDYM equations, including the matrix-valued Sparling equation, the use of Lax pairs, the application of Bäcklund transformations to known seed solutions, and of most importance to this work, the twistor construction where twistor data is freely prescribed and the solution of a classical Riemann–Hilbert (RH) problem is required in order to obtain the anti-self-dual field [11-13]. While various correspondences between these different methods have been known for some time, recently it has been shown that most can be directly obtained from a single construction [14].

Even though the ASDYM equations are in some technical sense solvable, obtaining explicit solutions given arbitrary data is difficult at best. It has been