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Large Deviation Estimates in the Stochastic Quantization of φ_2^4

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Abstract. We study in the small noise limit the behaviour of field trajectories for the process constructed by the authors in connection with the stochastic quantization of φ_2^4. Due to the presence of infinite renormalization the usual large deviation techniques do not apply immediately and a new strategy has to be developed. We prove some estimates analogous to the Freidlin–Ventzel inequalities. From these it follows that the field trajectories suitably smeared in space over a scale r_0 behave, when the noise is small, as the projection on the same scale of a field obeying a regularized stochastic equation with a large cut-off. However the estimates are not uniform in the cut-off and an interesting feature of the problem is that the scale over which the field is smeared determines whether the noise is sufficiently small for the estimates to apply.

0. Preliminaries

There exists a well developed theory of small random perturbations of dynamical systems evolving in R^n or on some finite dimensional manifold. This goes under the name of Freidlin–Ventzel theory[1] as these authors developed several basic ideas in this domain. Their fundamental estimates turned out to be equivalent to large fluctuation results of Varadhan[7]. The Freidlin–Ventzel approach was then extended to stochastic nonlinear partial differential equations of parabolic type in one space dimension besides time[2]. This extension of the F-V estimates follows from a careful but otherwise straightforward adaptation of the arguments developed for the finite dimensional case. The situation is entirely different if the number of space dimensions D is greater than 1. The prototype of equations we want to consider is

$$\frac{\partial \varphi}{\partial \tau} = \Delta\varphi - \varphi - V'(\varphi) + \varepsilon\frac{\partial W}{\partial \tau}, \tag{0.1}$$

where $\partial W/\partial \tau$ is a white noise in all variables. $V(\varphi)$ is an even polynomial in φ. These equations may be called stochastic Landau–Ginzburg equations. For $D \geqq 2$

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