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On Maximal Surfaces in Asymptotically Flat Space-Times

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Abstract. Existence of maximal and "almost maximal" hypersurfaces in asymptotically flat space-times is established under boundary conditions weaker than those considered previously. We show in particular that every vacuum evolution of asymptotically flat data for the Einstein equations can be foliated by slices maximal outside a spatially compact set and that every (strictly) stationary asymptotically flat space-time can be foliated by maximal hypersurfaces. Amongst other uniqueness results, we show that maximal hypersurfaces can be used to "partially fix" an asymptotic Poincaré group.

1. Introduction

The significant role played by maximal hypersurfaces in general relativity is well known and hardly needs to be discussed (cf. e.g. [An, CBY, COM]). A few years ago one of us [Ba1] established existence of such hypersurfaces under some interior regularity conditions together with some rather strong asymptotic conditions on the metric. In [Ba1] it was assumed that the metric tends to the flat metric as r^{-1} and the trace of the extrinsic curvature of the $t=\text{const}$ slices falls off at least as r^{-3}. In a recent analysis of solutions of the Einstein equations in which one can make global Lorentz transformations (the boost theorem) [COM] it was shown that for any $\alpha > 0$ there exists many regular solutions to the vacuum field equations for which the metric only falls off to the flat metric as $r^{-\alpha}$ and the extrinsic curvature as $r^{-1-\alpha}$. For such solutions, we cannot a priori expect the trace of the extrinsic curvature K to fall off faster than $r^{-1-\alpha}$. In this paper we show that in some situations even with such a slow fall-off the existence of maximal slices can be established, provided $\alpha > 1/2$. It may be of some interest to note that this decay condition covers, roughly speaking, all cases in which the mass has been shown to be finite and well defined ([Ba3, Ch2, OM]).

It has been recently observed [Br, Wi] that there exists a topological obstruction

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