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Intertwining Operators for Solving Differential Equations, with Applications to Symmetric Spaces

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Abstract. The use of intertwining operators to solve both ordinary and partial differential equations is developed. Classes of intertwining operators are constructed which transform between Laplacians which are self-adjoint with respect to different non-trivial measures. In the two-dimensional case, the intertwining operator transforms a non-separable partial differential operator to a separable one. As an application, the heat kernels on the rank 1 and rank 2 symmetric spaces are constructed.

It has long been appreciated that one of the nice properties of the special functions is that there exist differential operators which transform between functions of the same type, changing the indices of the functions by integral amounts. This property is the source of formulae which provide a compact expression for certain special functions in terms of powers of a differential operator applied to elementary functions. It has also been known for some time that the use of fractional differential operators, or pseudo-differential operators, extends the set of transformations between functions, allowing the indices to be changed by non-integral amounts.

Most of this common knowledge is for orthogonal polynomials and other special functions in one dimension. Similar results have been found for orthogonal polynomials in two dimensions [1, 2]. For particular coefficients, these two-dimensional orthogonal polynomials correspond to eigenfunctions of the radial part of the Laplace–Beltrami operator on certain rank 2 symmetric spaces. We present here a new approach to the construction and transformation among eigenfunctions of differential equations based on the construction of intertwining operators. This approach generalizes the classical operator transformations among special functions and is naturally applied in higher dimensions.

The intertwining operator approach to eigenfunctions was motivated by the method used by Dowker [3] to find the heat kernel for a free particle propagating on a Lie group manifold. It was developed by one of the authors [4] to find the heat kernel of a free-particle propagating on an n-dimensional sphere. Intertwining operators have also been used to solve non-linear integrable systems [5]. The intent here is to construct a few general classes of intertwining operators for one