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Ergodic Properties of a Kicked Damped Particle

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Abstract. We investigate a class of nonlinear dynamical systems describing the movement of a particle in a viscous medium under the influence of a kick force. These systems can be regarded as a generalization of the Langevin approach to Brownian motion in the sense that the fluctuating force on the particle is not Gaussian white noise but an arbitrary non-gaussian process generated by a nonlinear dynamical system. We investigate how certain properties of the force (periodicity, ergodicity, mixing property) transfer to the velocity of the particle. Moreover, the relaxation properties of the system are analysed.

1. Introduction

Consider a particle of mass 1 moving under the influence of the kick force

$$L_\tau(t) = \sum_{n=0}^{\infty} \eta_n \delta(t - n\tau). \tag{1}$$

τ is the time difference between subsequent kicks (for simplicity we assume that the impulses are equidistant). η_n is the strength of the kick at time $n\tau$. If the particle moves in a viscous medium, then in addition a friction force is exerted on the particle. We assume that this is proportional to the velocity $Y(t)$ of the particle. Thus the equation of motion is

$$\dot{Y} = -\gamma Y + L_\tau(t), \tag{2}$$

where γ is the viscosity of the liquid. We will keep our considerations as general as possible and allow $Y(t)$ to take values in $\mathbf{R}^m$. Moreover, we assume that there is a discrete time dynamical system $T: X \to X \subset \mathbf{R}^k$ and a function $f: X \to \mathbf{R}^m$ such that

$$\eta_n = f(x_{n-1}), \tag{3}$$

$$x_{n+1} = Tx_n. \tag{4}$$

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