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Harmonic Analysis of Local Operators

Detlev Buchholz

II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149,
D-2000 Hamburg 50, Federal Republic of Germany

Abstract. The spatial Fourier transforms of local operators are analysed. It is shown that the Fourier components for non-zero momentum form weakly square integrable functions in all states of finite energy. Moreover, there hold uniform bounds for the respective L^2-norms. The relevance of this result is illustrated in collision theory.

1. Introduction

The interplay between locality and the spectrum condition is one of the basic ingredients in many investigations of quantum field theory. The link between these structures is provided by harmonic (Fourier) analysis. It is the aim of the present paper to exhibit regularity properties of the spatial Fourier transforms of local operators which have escaped observation so far. These regularity conditions greatly simplify the analysis, notably in collision theory.

The setting and the notation used in this paper are standard. Let $\mathscr{H}$ be a Hilbert space, let $\mathfrak{A} \subset \mathscr{B}(\mathscr{H})$ be a *-algebra of local operators, and let U be a continuous unitary representation of the space-time translations $x \in \mathbb{R}^{s+1}$ which acts on $\mathscr{H}$ and satisfies the relativistic spectrum condition [1]. We recall that the condition of locality implies that for each operator $A \in \mathfrak{A}$ there exists some finite distance $d > 0$ such that

$$[A(x), A^*] = 0 \quad \text{if} \quad |\mathbf{x}| \geq |x_0| + d. \tag{1}$$

Here $\mathbf{x}$, x_0 denote the space and time part of the translation x with respect to a fixed coordinate system and we have introduced the notation

$$B(x) = U(x) B U(x)^{-1} \quad \text{for} \quad B \in \mathscr{B}(\mathscr{H}). \tag{2}$$

We write $B(\mathbf{x})$ if the time component x_0 of x in relation (2) is zero, and similarly $B(x_0)$ if $\mathbf{x} = 0$.