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A Dynamical System with Integer Information Dimension and Fractal Correlation Exponent*

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Abstract. In this paper we construct a family $\{T_\gamma\}$, $0<\gamma<\frac{1}{2}$, of exact endomorphisms of $[0,1]$ such that the invariant measure m_γ of T_γ is equivalent to Lebesgue measure but has fractal correlation exponent $\nu=2\gamma$. This shows that an almost complete dichotomy can exist between the information dimension and the correlation exponent in observable dynamical systems.

1. Introduction

In the study of dynamical systems possessing an observable attractor A and ergodic occupation measure m two basic notions of dimension have developed. One is that of the fractal dimension of the attractor itself viewed as a geometrical object in space. The second is a measure-dependent notion which incorporates the actual dynamics of the system by taking into account the relative frequency with which a typical orbit visits different regions of the attractor. However even if we confine ourselves to one of these basic approaches the definition of dimension is not unique (see Farmer, Ott, and Yorke [3] and Hentschel and Procaccia [6]) although the various possible definitions may coincide in special cases. The fractal dimension D of the attractor A may be taken to be the Hausdorff dimension of A or perhaps the capacity of A. Among measure-dependent notions the information dimension σ and the correlation exponent ν (sometimes also called correlation dimension) have been widely discussed. Grassberger and Procaccia [4] introduced the correlation exponent as a measure-dependent gauge of dimension possessing the attractive feature of being easy to compute numerically from time series data. It was recognized immediately (Grassberger and Procaccia [4, 5] and Hentschel and Procaccia [6]) that for a large class of fractal attractors the general inequalities $\nu\leq\sigma\leq D$ hold with simultaneous equality $\nu=\sigma=D$ occurring if and only if the distribution is uniform across the attractor. However, in the estimation of dimension of a dynamical system from time series data, it appears to be a commonly held view that ν and σ are generally close although not identical, and

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