Commun. Math. Phys. 129, 599–619 (1990)

Communications in Mathematical Physics

Weak Matching Rules for Quasicrystals

Joshua E. S. Socolar

Department of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. Weak matching rules for a quasicrystalline tiling are local rules that ensure that fluctuations in "perp-space" are uniformly bounded. It is shown here that weak matching rules exist for N-fold symmetric tilings, where N is any integer not divisible by four. The result suggests that, contrary to previous indications, quasicrystalline ground states are not confined to those symmetries for which the incommensurate ratios of wavevectors are quadratic irrationals. An explicit method of constructing weak matching rules for N-fold symmetric tilings in two dimensions is presented. It is shown that the generalization of the construction yields weak matching rules in the case of icosahedral symmetry as well.

1. Introduction

The discovery of quasicrystalline phases of certain alloys raises a fundamental question in the theory of solid structure. Icosahedral and decagonal quasicrystals have been observed, as have octagonal and dodecagonal samples with a lesser degree of translational order, but no other noncrystallographic symmetries have been observed to date. Is there some a priori principle that prohibits them?

A useful way to approach the question is to represent quasicrystals as space-filling arrangements of rigid tiles of two or more types, where it is assumed that a good approximation to the actual atomic structure is obtained by decorating each tile of a given type with the same atomic motif. It is well known that quasicrystalline tilings of arbitrary symmetry can be constructed using a variety of algorithms [1]. The mere existence of a tiling does not imply, however, that it is a plausible template for a real atomic structure. It may be that certain tilings or even entire symmetry classes can be eliminated from consideration on physical grounds.

One criterion for physical relevance is locality. In real solids, the interaction between atoms decays rapidly with their separation, so a physically plausible model Hamiltonian should not contain terms depending on the relative positions of widely separated units. In a tiling model, where the energetics are encoded in