# A "Transversal" Fundamental Theorem
# for Semi-Dispersing Billiards

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*Dedicated to Joel L. Lebowitz on the occasion of his 60th birthday*

**Abstract.** For billiards with a hyperbolic behavior, Fundamental Theorems ensure an abundance of geometrically nicely situated and sufficiently large stable and unstable invariant manifolds. A "Transversal" Fundamental Theorem has recently been suggested by the present authors to prove *global ergodicity* (and then, as an easy consequence, the K-property) of semi-dispersing billiards, in particular, the global ergodicity of systems of $N \geq 3$ elastic hard balls conjectured by the celebrated *Boltzmann-Sinai ergodic hypothesis*. (In fact, the suggested "Transversal" Fundamental Theorem has been successfully applied by the authors in the cases $N = 3$ and 4.) The theorem generalizes the Fundamental Theorem of Chernov and Sinai that was really the fundamental tool to obtain *local ergodicity* of semi-dispersing billiards. Our theorem, however, is stronger even in their case, too, since its conditions are simpler and weaker. Moreover, a complete set of conditions is formulated under which the Fundamental Theorem and its consequences like the Zig-zag theorem are valid for general semi-dispersing billiards beyond the utmost interesting case of systems of elastic hard balls. As an application, we also give conditions for the ergodicity (and, consequently, the K-property) of dispersing-billiards. "Transversality" means the following: instead of the stable and unstable foliations occurring in the Chernov-Sinai formulation of the stable version of the Fundamental Theorem, we use the stable foliation and an arbitrary nice one transversal to the stable one.

## 1. Introduction

Smooth dynamical systems with singularities satisfying a hyperbolicity condition play an utmost important role in the theory of dynamical systems for (i) they contain such interesting classes as systems of elastic hard balls or – more generally