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The $\bar{\partial}$-Operator on Algebraic Curves

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Abstract. For a singular algebraic curve we show that all closed extensions of $\bar{\partial}$ are Fredholm, and we give a general index formula. In particular, we prove a modified version of a conjecture due to MacPherson.

1. Introduction

Let M be a Kähler manifold of complex dimension m and denote by $\Omega^{p,q}(M)$ and $\Omega_0^{p,q}(M)$ the space of smooth complex valued forms of type (p, q) on M and the subspace of forms with compact support, respectively. The Dolbeault complex

$$0 \to \Omega_0^{0,0}(M) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \Omega_0^{0,m}(M) \to 0 \tag{1.1}$$

is well known to be elliptic. If M is compact the cohomology, $H^{0,*}(M)$, is finite and the index,

$$\chi(M) := \sum_{q \geq 0} (-1)^q \dim H^{0,q}(M), \tag{1.2}$$

is called the arithmetic genus of M (cf. [H]). If M is not compact one can use the Hilbert space structure induced by the metric to define

$$\Omega_{(2)}^{p,q}(M) := \left\{ \omega \in \Omega^{p,q}(M) \mid \int_M \omega \wedge *\omega < \infty, \int_M \bar{\partial}\omega \wedge *\bar{\partial}\omega < \infty \right\}. \tag{1.3}$$

Here the Hodge $*$ operator on real forms is extended as an antilinear map. This leads to another complex

$$0 \to \Omega_{(2)}^{0,0}(M) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \Omega_{(2)}^{0,m}(M) \to 0, \tag{1.4}$$

the cohomology of which is called the L^2-$\bar{\partial}$-cohomology, $H_{(2)}^{0,*}(M)$. It is natural to ask for conditions on M which ensure the finiteness of $H_{(2)}^{0,*}(M)$. If this is