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Integration on the n^{th} Power of a Hyperbolic Space in Terms of Invariants Under Diagonal Action of Isometries (Lorentz Transformations)

Bent Fuglede

Matematisk Institut, Københavns Universitet, Universitetsparken 5, DK-2100 København Ø, Denmark

Abstract. The integral of a function over the n'th power of hyperbolic d-dimensional space H is decomposed into integration along each orbit under diagonal action on H^n of the isometry group G on H, followed by integration over the orbit space, parametrized in terms of a complete set of invariants. The Jacobian entering in this last integral is expressed explicitly in terms of certain determinants. When viewing H as a half-hyperboloid in $\mathbb{R}^{d+1}$, G is induced by the homogeneous Lorentz group $O^{\uparrow}(1,d)$ acting on $\mathbb{R}^{d+1}$.

Contents

Introduction

We shall consider a problem which enters in connection with the study of tensor products of certain representations of the Poincaré group and the Lorentz group. These particular representations occur for example in the study of a free scalar quantum field in Fock space, see e.g. Bogoliubov, Logunov and Todorov [1].

The problem is to give an explicit formula for the decomposition of an integral over the n^{th} power of hyperbolic d-dimensional space into integrals along the orbits under diagonal action of the isometry group G on that space followed by an integral over the orbit space, parametrized in terms of a complete set of invariants.

It can be shown that there is an integral formula of this kind in the case of