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Hydrodynamic Limit for a System with Finite Range Interactions

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Abstract. We study a system of interacting diffusions. The variables present the amount of charge at various sites of a periodic multidimensional lattice. The equilibrium states of the diffusion are canonical Gibbs measures of a given finite range interaction. Under an appropriate scaling of lattice spacing and time, we derive the hydrodynamic limit for the evolution of the macroscopic charge density.

1. Introduction

The derivation of the hydrodynamic equation for infinite particle systems with conservation law has been the subject of active research. One such model is the *Ginzburg–Landau* model [11]. The hydrodynamic equation for this model is obtained in [3] and [4]. In this model charges are located at the various sites of a periodic multidimensional lattice. The flow of these charges from one site to another is governed by a suitable diffusion law. After an appropriate space and time scaling, the microscopic charge density converges to a deterministic limit which is characterized as the solution of a nonlinear parabolic equation.

The passage to the hydrodynamic limit for the Ginzburg–Landau model under certain conditions was studied in [4]. We describe these conditions.

For any positive integer N, let S_N denote the periodic lattice $\{j: j=0,1,\dots,N\}$ with 0 and N identified, and let S_N^d denote the product of d copies of S_N. For each site a in S_N^d, there is a random variable $x_a = x_a(t)$ which is the amount of charge at site a. The family of x_a undergo a diffusion with generator

$$\mathscr{L}_N^0 = \frac{N^2}{2}\left[\sum\left(\frac{\partial}{\partial x_a}-\frac{\partial}{\partial x_b}\right)^2 - \sum(\phi'(x_a)-\phi'(x_b))\left(\frac{\partial}{\partial x_a}-\frac{\partial}{\partial x_b}\right)\right], \tag{1.1}$$

where both sums are over the adjacent sites a and b in S_N^d. The generator $\mathscr{L}_N^0$ is formally symmetric with respect to the product measure $\rho_N(d\underline{x})$ defined as

$$\rho_N(d\underline{x}) = \prod_{a\in S_N^d} e^{-\phi(x_a)}dx_a, \tag{1.2}$$