

Global Flows with Invariant (Gibbs) Measures for Euler and Navier–Stokes Two Dimensional Fluids

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Abstract. We construct a family of probability spaces $(\Omega, \mathcal{F}, P_\gamma)$, $\gamma > 0$ associated with the Euler equation for a two dimensional inviscid incompressible fluid which carries a pointwise flow ϕ_t (time evolution) leaving P_γ globally invariant. ϕ_t is obtained as the limit of Galerkin approximations associated with Euler equations. P_γ is also in invariant measure for a stochastic process associated with a Navier–Stokes equation with viscosity γ , stochastically perturbed by a white noise force.

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0. Introduction

The search for solutions of Euler and Navier–Stokes equations in certain spaces of functions is of great physical and mathematical interest. In the present paper we shall discuss the Euler and Navier–Stokes equations for an incompressible fluid.

For simplicity we shall consider a fluid confined in a rectangular box \mathbb{T}^2 , with periodic boundary conditions (or, equivalently, on a 2-torus), but extensions to