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Universal Schwinger Cocycles of Current Algebras in $(D+1)$-Dimensions: Geometry and Physics

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Abstract. We discuss the universal version of the Schwinger terms of current algebra (we call it the universal Schwinger cocycle) for $p=3$ (here p denotes the class of the Schatten ideal I_p, which is related to the $(D+1)$ space-time dimensions by $p=(D+1)/2$) in detail, and give a conjecture of the general form of the cocycle for any p. We also discuss the infinite charge renormalizations, the highest weight vector and state vectors for $p=3$. Last, we give brief comments on the problems caused by the difficulties to construct the measure of infinite-dimensional Grassmann manifolds.

1. Introduction

In particle physics, current algebra has been introduced in the study of strong interactions. It was assumed that the time-component of a current generates a closed algebra in the classical level. More explicitly, we consider a Dirac field in $D+1$-dimension coupled to an external Yang–Mills field A. Let G be a compact semi-simple Lie group and $\underline{g}$ its algebra. The current is

$$J^i(x) = \Psi^\dagger(x)\lambda^i\Psi(x). \tag{1}$$

We define

$$J(f) \overset{\text{def}}{=} \int dx f^i(x) J^i(x), \tag{2}$$

where $f(x) = f^i(x)\lambda^i : X \to \underline{g}$, is a mapping valued in the Lie algebra.

This operator satisfies

$$[J(f), J(g)] = J([f,g]). \tag{3}$$

But, in the quantum level, this relation is modified as follows:

$$[J(f), J(g)] = J([f,g]) + c(f,g;A). \tag{4}$$

This $v(f,g;A)$ is called the Schwinger term [F]. This requires the representations of the Abelian extension $\widehat{\mathrm{Map}(X;\underline{g})}$ of $\mathrm{Map}(X;\underline{g})$,

$$(0 \to \mathrm{Map}(A;C) \to \widehat{\mathrm{Map}(X;\underline{g})} \to \mathrm{Map}(X;\underline{g}) \to 0).$$