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Temperature Correlators of the Impenetrable Bose Gas as an Integrable System

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Abstract. It is shown that the temperature equal-time correlators of impenetrable bosons in one space dimension are described by a classical integrable system. Partial differential equations for two-point as well as for multipoint correlators are obtained. The short-distance and low-density expansions are constructed.

1. Introduction

The Hamiltonian of the one-dimensional non-relativistic Bose gas [1] is

$$H=\int_{-\infty}^{\infty}(\partial_z\psi^+\partial_z\psi+c\psi^+\psi^+\psi\psi-h\psi^+\psi)dz\,. \tag{1.1}$$

Here $\psi(z)$, $\psi^+(z)$ are canonical Bose fields, $[\psi(z),\psi^+(y)]=\delta(z-y)$ and h is a chemical potential. Only the case of impenetrable bosons is considered below, the corresponding value of the coupling constant being $c=+\infty$. The thermodynamics of the model was constructed in paper [2]. At zero temperature the thermal equilibrium state is the ground state of the Hamiltonian, representing a Fermi zone. All the states of particles with momenta k, $-q\leqq k\leqq q$ are filled (here $q=h^{1/2}$ is the Fermi momentum). At temperature $T>0$, the thermal equilibrium distribution of particles is given by the Fermi weight

$$w(k,h,T)=(1+\exp\{\varepsilon(k)/T\})^{-1}\,, \tag{1.2}$$

where $\varepsilon(k)=k^2-h$ is a particle energy. Gas density D is

$$D=\frac{1}{2\pi}\int_{-\infty}^{\infty}w(k,h,T)dk \tag{1.3}$$

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