Commun. Math. Phys. 129, 169–204 (1990)

Communications in
Mathematical
Physics

On the Free Boundary Problem for Stationary Compressible Navier-Stokes Equations

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Abstract. We consider the equations which describe a stationary motion of a viscous compressible barotropic fluid in a bounded domain in $\mathbb{R}^3$ with a free boundary determined by the surface tension. By means of some a priori estimates we prove the existence of rotationally symmetric solutions (in reality with some additional symmetry) for a sufficiently small external force and in the case of rotationally symmetric force and domain (where also we need more symmetry, respectively).

1. Introduction

In this paper a free boundary problem for a viscous compressible barotropic fluid is considered. The stationary motion of the fluid in a bounded domain $\Omega \subset \mathbb{R}^3$ with a free boundary S is described by the following equations [24]:

$$\begin{aligned} \varrho v \nabla v + \nabla p + A v &= \varrho f \quad \text{in } \Omega , \\ \operatorname{div}(v\varrho) &= 0 \quad \text{in } \Omega , \\ v \cdot \bar{n} &= 0 \quad \text{on } S , \\ T\bar{n} - (\bar{n} T \bar{n})\bar{n} &= 0 \quad \text{on } S , \end{aligned} \tag{1.1}$$

where $Av = -\mu\Delta v - \nu\nabla \operatorname{div} v$, $v = v(x)$ is the velocity of the fluid, $\varrho = \varrho(x)$ the density y, $p = p(\varrho)$ the pressure (which is a given function of ϱ), $f = f(x)$ the external force field per unit mass. The viscosity coefficients μ and ν satisfy the thermodynamic restrictions

$$\mu > 0 , \quad \nu \geqq \tfrac{1}{3}\mu . \tag{1.2}$$

We also use the deformation tensor T with elements

$$T_{ij} = -p\delta_{ij} + T'_{ij}(v) , \quad i,j = 1,2,3 , \tag{1.3}$$