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Nuclear Maps and Modular Structures II: Applications to Quantum Field Theory

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Abstract. A correspondence between spectral properties of modular operators appearing in quantum field theory and the Hamiltonian is established. It allows to prove the "distal" split property for a wide class of models. Conversely, any model having this property is shown to satisfy the Haag–Swieca compactness criterion. The results lead to a new type of nuclearity condition which can be applied to quantum field theories on arbitrary space-time manifolds.

1. Introduction

The physical significance of the modular operators appearing in quantum field theory is still a mystery: given the von Neumann algebra $\mathcal{A}(\mathcal{O})$ of operators which are associated with a space-time region $\mathcal{O}$ and given the vector Ω representing the vacuum state, the Tomita–Takesaki theory [3] provides us on the one hand with a one-parameter group of unitaries $\Delta^{it}, t\in\mathbb{R}$ (the modular group) which induces automorphisms of $\mathcal{A}(\mathcal{O})$. Moreover, the restriction of the vacuum state to $\mathcal{A}(\mathcal{O})$ behaves like an equilibrium state under the action of these automorphisms (it satisfies the KMS-condition). These mathematical facts seem to suggest that the modular group is some kind of dynamics of an observer in the region $\mathcal{O}$.

On the other hand, all attempts to justify such a physical interpretation in general have failed so far. There is only a single, though important special case where this interpretation is known to be correct: if $\mathcal{O}$ is a wedge-shaped region which is bounded by two characteristic planes, it can be shown that the corresponding modular group is the dynamics of a uniformly accelerated observer [21]. But for arbitrary regions such a clearcut interpretation is out of sight.

As a step towards a clarification of this point we exhibit in the present investigation a tight relation between spectral properties of the modular operators and the Hamiltonian H. Although our results do not solve the problem of the

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