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Relating Microscopic and Macroscopic Parameters for a 3-Dimensional Random Walk*

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Abstract. We consider a particle undergoing a discrete random walk with killing. We relate the microscopic transition and killing probabilities to these same parameters at a macroscopic level. We find the appropriate scaling laws.

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Introduction

Consider the unit cube $\mathbb{C}$ in three-dimensional space. For any value of $n = 0, 1, 2, \ldots$ we partition this unit cube $\mathbb{C}$ into 8^n "little cubes" denoted by $\mathbb{C}_n$. These cubes are obtained by successive bisections of each of the sides of the unit cube and the sides of each $\mathbb{C}_n$ has length $(1/2)^n$.

For a fixed value of n we consider a 2-step Markov process with state space given by the 8^n "little cubes" $\mathbb{C}_n$. The evolution of a "particle" in this discrete time process is as follows: at each site there is a probability v_n of being killed. If a particle is not killed at a site $\mathbb{C}_n$, then it makes a transition to one of its six neighbors with probabilities that depend on the way in which the particle arrived at the present state. These probabilities are f_n, b_n and s_n and they give the probability of a "forward transition," i.e. one that preserves the direction of the last transition, a "backward transition." i.e. one that reverses this direction or finally a "sideways transition." where a ninety degree turn (in any one of the four possible directions) with respect

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