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Adiabatic Theorems for Dense Point Spectra*

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Abstract. We prove adiabatic theorems in situations where the Hamiltonian has dense point spectrum. The gap condition of the standard adiabatic theorems is replaced by an appropriate condition on the ineffectiveness of resonances.

1. Introduction

The prototype Adiabatic Theorem in quantum mechanics asserts that for operators with discrete spectra, in the adiabatic limit, the physical evolution takes an instantaneous eigenstate at $t = 0$ to the corresponding instantaneous eigenstate at a later time [3, 8]. More generally [1], with no assumptions about the nature of the instantaneous spectrum, but provided it has a gap for all times, the physical evolution in the adiabatic limit respects the splitting of the Hilbert space into spectral subspaces: A state in the subspace below the gap at $t = 0$ will evolve to a state that lies in the corresponding subspace below the corresponding gap at time t. While there are various kinds of adiabatic theorems that deal with the two settings above, (i.e. discrete spectra or gap conditions [1–3, 6, 8–11]), there are no results for situations that have no gaps, in particular in cases that involve dense point spectra. Our purpose here is to describe such results.

There is actually a good physical reason for the gap condition. A time-dependent Hamiltonian $H(t/\tau)$ with time scale τ, can be thought of as describing a quantum system in an external time-dependent field, that for the sake of discussion we call photons. The photon field is switched on in the distant past and switched off in the distant future and in the limit $\tau \to \infty$ contains only soft photons with frequencies characterized by $1/\tau$. These cannot excite the system when it has gaps and that is why the evolution respects the spectral structure even on long time scales of order

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