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Chern-Simons Gauge Theory and Projectively Flat Vector Bundles on $\mathcal{M}_g$

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Abstract. We consider a vector bundle on Teichmüller space which arises naturally from Witten's analysis of Chern-Simons Gauge Theory, and define a natural connection on it. In the case when the gauge group is $U(1)$ we compute the curvature, showing, in particular, that the connection is projectively flat.

1. Introduction

Projectively flat unitary vector bundles on the moduli space of curves are well-known to define conformal field theories [FS]. When such a bundle $\mathcal{W}$ is pulled back to Teichmüller space (which is simply connected), it has a finite-dimensional space $\mathcal{H}_{\mathcal{W}}$ of projectively flat sections with the dimension of $\mathcal{H}_{\mathcal{W}}$ being equal to the rank of $\mathcal{W}$. Moreover, the modular group acts projectively on the space of these sections. In [W], it was argued that the state-space of Chern-Simons gauge theory is, up to projective isomorphism, given by $\mathcal{H}_{\mathcal{W}}$ for a certain projectively flat bundle. The bundle $\mathcal{W}$ is well-known; our aim in this note is to give a purely differential-geometric description of a natural connection on $\mathcal{W}$, with the correct covariance properties under the modular group. The definition draws on a construction from [RSW], and further, makes a certain technical assumption, namely, that the state-vectors of the Chern-Simons theory are normalisable. In the case when the gauge group is $U(1)$, we prove the projective flatness of this connection in a computation which also yields the "central charge." (In this case the assumption of finite norm is trivially true, as we shall see.) It appears that this may complement the deeper treatments of [BN, EMSS] which however involve considerations from conformal field theory.

In Sect. 4 we outline a short proof of projective flatness, again in the $U(1)$ case, which uses the "discrete Heisenberg group."

We are not able, at this point, to prove the projective flatness of the connection for nonabelian groups.

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