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Extremity of the Disordered Phase in the Ising Model on the Bethe Lattice

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Abstract. We prove that the disordered Gibbs distribution in the ferromagnetic Ising model on the Bethe lattice is extreme for $T \geqq T_c^{SG}$, where T_c^{SG} is the critical temperature of the spin glass model on the Bethe lattice, and it is not extreme for $T < T_c^{SG}$.

1. Introduction

The Bethe lattice $\mathscr{T}^k$ of degree $k \geqq 1$ is a tree (i.e. a graph without cycles) such that exactly $(k+1)$ edges come out from any of its vertex. The Ising model on the Bethe lattice is defined by the Hamiltonian

$$H(\sigma) = -\sum_{\langle x, y \rangle} J_{xy} \sigma(x) \sigma(y), \tag{1.1}$$

where the sum is taken over all pairs of the nearest neighbors $\langle x, y \rangle$ and the spins $\sigma(x)$ take values ± 1.

In the ferromagnetic Ising model

$$J_{xy} \equiv J > 0, \tag{1.2}$$

and in the spin glass model the interaction J_{xy} is random and

$$J_{xy} = \pm J, \quad J > 0, \tag{1.3}$$

with probability 1/2 *independently for any pair* $\langle x, y \rangle$. Both in the ferromagnetic Ising model and in the spin glass model phase transitions occur, but the values of the corresponding critical temperatures are different. Denote

$$\theta = \tanh(J/T). \tag{1.4}$$

Then the critical value θ for the ferromagnetic Ising model is

$$\theta_c^F = 1/k \tag{1.5}$$