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Quantum Evolution and Classical Flow in Complex Phase Space

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Abstract. For a class of holomorphic perturbations of the harmonic oscillator in n degrees of freedom a local solution of the time-dependent Schrödinger equation in the Bargmann representation is constructed which pointwise propagates, to leading order in $\hbar$, along the classical trajectories in complex phase space.

I. Introduction and Statement of the Result

The relation between the quantum flow, i.e. the solutions of the time-dependent Schrödinger equation, and the corresponding classical Hamiltonian flow is a very old problem of quantum mechanics. It is well known that the strongest possible relation between classical and quantum flow, (in the sense that the classical evolution determines the quantum one *exactly*, not just at leading order in h, and *pointwise*, not just in L^2 sense) takes place for the coherent states of a system of linear oscillators, i.e. for a system of linear oscillators provided their quantum evolution is described in the Bargmann representation of the canonical commutation rules.

Consider indeed, for $(q,p)\in T^*\mathbf{R}^n \cong \mathbf{R}^{2n}$, $\{p_i, q_j\}=\delta_{ij}$, the classical Hamiltonian of a system of n independent oscillators of unit frequencies:

$$H_0(p,q)=\tfrac{1}{2}\sum_{k=1}^{n}(p_k^2+q_k^2). \tag{1.1}$$

The transformation:

$$\begin{aligned} C: z_k &= \frac{1}{\sqrt{2}}(q_k-ip_k),\ \bar{z}_k=\frac{1}{\sqrt{2}}(q_k+ip_k) \\ C^{-1}: q_k &= \frac{1}{\sqrt{2}}(z_k+\bar{z}_k),\ p_k=\frac{1}{\sqrt{2}i}(\bar{z}_k-z_k) \end{aligned} \tag{1.2}$$