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Mean-Field Critical Behaviour for Percolation in High Dimensions

Takashi Hara[1,*] and Gordon Slade[2,**]

[1] Courant Institute of Mathematical Sciences, N.Y.U., 251 Mercer Street, New York, NY 10012, USA

[2] Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S4K1

Abstract. The triangle condition for percolation states that $\sum_{x,y} \tau(0,x)\,\tau(x,y)\cdot\tau(y,0)$ is finite at the critical point, where $\tau(x,y)$ is the probability that the sites x and y are connected. We use an expansion related to the lace expansion for a self-avoiding walk to prove that the triangle condition is satisfied in two situations: (i) for nearest-neighbour independent bond percolation on the d-dimensional hypercubic lattice, if d is sufficiently large, and (ii) in more than six dimensions for a class of "spread-out" models of independent bond percolation which are believed to be in the same universality class as the nearest-neighbour model. The class of models in (ii) includes the case where the bond occupation probability is constant for bonds of length less than some large number, and is zero otherwise. In the course of the proof an infrared bound is obtained. The triangle condition is known to imply that various critical exponents take their mean-field (Bethe lattice) values ($\gamma=\beta=1, \delta=\Delta_t=2, t\geqq 2$) and that the percolation density is continuous at the critical point. We also prove that $\nu_2=1/2$ in (i) and (ii), where ν_2 is the critical exponent for the correlation length.

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* Supported by the Nishina Memorial Foundation and NSF grant PHY-8896163. Address after September 1, 1989: Department of Mathematics, University of Texas at Austin, Austin, TX 78712, USA. Electronic address: hara @ emx. utexas. edu

** Supported by NSERC grant A9351. Electronic address: slade @ sscvax. mcmaster. ca