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# The Pressure in the Huang–Yang–Luttinger Model of an Interacting Boson Gas

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**Abstract.** This completes our study of the equilibrium thermodynamics of the Huang–Yang–Luttinger model of a boson gas with a hard-sphere repulsion. In an earlier paper we obtained a lower bound on the pressure, but our proof of an upper bound held only for a truncated version of the model. In this paper we establish an upper bound on the pressure in the full model; the upper and lower bounds coincide and provide a variational formula for the pressure. The proof relies on recent second-level large deviation results for the occupation measure of the free boson gas.

## 1. Introduction

Huang, Yang and Luttinger [1] introduced a model of a boson gas with a hard-sphere repulsion which may be described thus: let $\Lambda_1, \Lambda_2, \ldots$ be a sequence of regions in $\mathbb{R}^d$ with $V_l$, the volume of $\Lambda_l$, tending to infinity with $l$; with each region $\Lambda_l$, we associate the sequence $\varepsilon_l(1) \leqq \varepsilon_l(2) \leqq \cdots$ of ordered real numbers interpreting $\varepsilon_l(j)$ as the $j^{th}$ eigenvalue of the single-particle Hamiltonian of the non-interacting system in the region $\Lambda_l$, so that the free-gas Hamiltonian $H_l^0$ is given by

$$H_l^0 = \sum_{j \geqq 1} \varepsilon_l(j) n_l(j), \tag{1.1}$$

where $n_l(j)$ is the occupation number of the $j^{th}$ level; then the Huang–Yang–Luttinger model is described by the Hamiltonian

$$H_l^{\mathrm{HYL}} = H_l^0 + \frac{a}{2V_l}\left\{2N_l^2 - \sum_{j \geqq 1} n_l(j)^2\right\}, \tag{1.2}$$

where $N_l = \sum_{j \geqq 1} n_l(j)$ is the total number of particles and $a > 0$. The physics of this model was discussed by Huang, Yang and Luttinger [1] and by Thouless [2] and reviewed in our recent paper [3]; we do not repeat the discussion here, except to recall that in [1] the authors argued that the condensate, if any, would occupy