Commun. Math. Phys. 128, 197–212 (1990)

Communications in
Mathematical
Physics

From Geometric Quantization to Conformal Field Theory

A. Alekseev and S. Shatashvili

Leningrad Steklov Mathematical Institute, Fontanka 27, SU-191011 Leningrad, USSR

Abstract. Investigation of $2d$ conformal field theory in terms of geometric quantization is given. We quantize the so-called model space of the compact Lie group, Virasoro group and Kac-Moody group. In particular, we give a geometrical interpretation of the Virasoro discrete series and explain that this type of geometric quantization reproduces the chiral part of CFT (minimal models, $2d$-gravity, WZNW theory). In the appendix we discuss the relation between classical (constant) r-matrices and this geometrical approach.

1. Introduction

In this paper we continue an investigation of $2d$ conformal field theories in terms of geometric quantization (see [1–3]). As demonstrated in our previous papers, the standard geometric quantization method [4] can be reformulated in terms of the path integral approach. In [2] the correspondence between the coadjoint orbit and the irreducible representation of compact Lie groups was explicitly realized by means of the functional integral. More precisely, we constructed in [2] a quantum mechanical system, such that the path integral with boundary conditions gives matrix coefficients of the corresponding irreducible representation. The action functional of this system is defined by the canonical symplectic structure Ω on the given coadjoint orbit and a Hamiltonian $H(X)$, which is a function on the orbit: $S=\int d^{-1}\Omega-\int H(X)dt$; this action is a functional of trajectories on the orbit. Later in [3] using the same rules, we described quantum field theory on the coadjoint orbit of infinite dimensional Lie groups (Virasoro, Kac-Moody) and the properties of the corresponding action functional investigated. In particular, we have shown that for the Virasoro group the geometrical action, written in terms of group variables $F(x)\in \operatorname{diff} S^1$ differs from the action in $2d$ gravity [5] by the extra term $\int b_0\dot{F}F'dxdt$, where the number b_0 parametrizes generic coadjoint orbits. (A similar statement is true also for Kac-Moody group and WZNW model.) In the language of geometric quantization the appearance of $SL(2,\mathbf{R})$ current algebra in $2d$ gravity is the consequence of symplectic geometry, and as it was shown in [3] Virasoro