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Affine Kac-Moody Algebras and Semi-Infinite Flag Manifolds

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Dedicated to Dmitry Borisovich Fuchs on his 50th birthday

Abstract. We study representations of affine Kac-Moody algebras from a geometric point of view. It is shown that Wakimoto modules introduced in [18], which are important in conformal field theory, correspond to certain sheaves on a semi-infinite flag manifold with support on its Schubert cells. This manifold is equipped with a remarkable semi-infinite structure, which is discussed; in particular, the semi-infinite homology of this manifold is computed. The Cousin-Grothendieck resolution of an invertible sheaf on a semi-infinite flag manifold gives a two-sided resolution of an irreducible representation of an affine algebra, consisting of Wakimoto modules. This is just the BRST complex. As a byproduct we compute the homology of an algebra of currents on the real line with values in a nilpotent Lie algebra.

1. Introduction

In [18, 19] we have introduced and studied a new class of representations of affine Kac-Moody algebras, the so-called *Wakimoto modules* [44]. These representation allow *bosonic realization*, the Sugawara energy-momentum tensor being quadratic in bosons. This gives a new bosonization rule for the Wess-Zumino-Witten (WZW) models. In [19] we explicitly constructed the intertwining operators between Wakimoto modules and chains (or primary fields) which are submodules of their homomorphisms, using vertex operators. Our results enable us to give an integral representation of the correlation functions in WZW models on the plane in spirit of [14] (it was done soon after [15, 27]). In [20] we have proposed the *two-sided Bernstein-Gelfand-Gelfand (BGG) resolution*, or BRST complex, of an irreducible representation of an affine Kac-Moody algebra, consisting of Wakimoto modules (recall that the usual BGG resolution [7, 26, 41] is one-sided and consists of Verma modules). According to Felder's work [23] (where a similar resolution is constructed over the Virasoro algebra), this

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