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Asymptotic Neutrality of Large Ions

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Abstract. It is proved that a nucleus of charge Z can bind at most $Z+O(Z^a)$ electrons, with $a=47/56$.

Consider the Hamiltonian for a nucleus of charge Z and N quantized electrons,

$$H_{Z,N}=\sum_{i=1}^{N}\left[(-\Delta_{x_i})-\frac{Z}{|x_i|}\right]+\frac{1}{2}\sum_{i\neq j}\frac{1}{|x_i-x_j|}=-\Delta+V_{\text{Coulomb}}.$$

The ground state energy is then

$$E(Z)=\inf_N E(Z,N)=\inf_N \inf_{\substack{\psi\in\mathscr{H}\\ \|\psi\|_2=1}} \langle H_{Z,N}\psi,\psi\rangle,$$

where $\mathscr{H}=\bigwedge_{i=1}^{N}(L^2(\mathbf{R}^3)\otimes\mathbf{C}^q)$ is the space of antisymmetric wave functions with q spins. Throughout this paper we will simply refer to them as "antisymmetric" wave functions.

For each Z, call $N(Z)$ the smallest number for which $E(Z)=E(Z,N)$. It is an interesting problem to obtain sharp estimates for $N(Z)$. The sharpest known result appears in [8], where the reader will find a discussion of the history of the problem. In particular, $N(Z)/Z\to 1$ as $Z\to\infty$, although there were no estimates for the rate of convergence. Our main result is the following:

Theorem.

$$N(Z)=Z+O(Z^\alpha)\quad\text{for}\quad \alpha=\frac{47}{56}.$$

We announced this result in [1]. We are grateful to V. Bach for pointing out a

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