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Area-Preserving Diffeomorphisms and Supermembrane Lorentz Invariance

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Abstract. The Lie algebra of area-preserving diffeomorphisms on closed membranes of arbitrary topology is investigated. On the basis of a harmonic decomposition we define the structure constants as well as two other tensors which appear in the supermembrane Lorentz generators. We derive certain identities between these tensors and analyze their validity when the area-preserving diffeomorphisms are approximated by $SU(N)$. One of the additional tensors can then be identified with the invariant symmetric three-index tensor of $SU(N)$, while the second has no obvious analog. We prove that the Lorentz generators are classically conserved in the light-cone gauge for arbitrary membrane topology, as a consequence of these tensor identities. This formulation allows a systematic study of the violations of Lorentz invariance in the $SU(N)$ approximation.

1. Introduction

The recent interest in (super)membrane theory has been motivated as much by their mathematical structure as by their possible interest for the unification of fundamental interactions (recent developments are reviewed in [1]). Of particular importance is the group of area-preserving (symplectic) diffeomorphisms which naturally appears in membrane theory as a residual symmetry in the light-cone gauge [2, 3] (the corresponding symmetry in string theory consists only of the "length-preserving" diffeomorphism $\sigma \to \sigma +$ constant, and is thus rather trivial). Little is known about these infinite-dimensional groups in general (for an early reference in the physics literature, see [4]), and one of the possible benefits of membrane theories could well be an improved understanding of their mathematical structure. A remarkable result, already obtained some time ago, is that, for spherical membranes, the structure constants of the group of area-preserving diffeomorphisms can be obtained as the $N \to \infty$ limit of the $SU(N)$ structure constants [2, 3]. More recently, and partly motivated by the discovery of supermembranes [5], several papers have appeared dealing with area-preserving diffeomorphisms on the sphere and the torus [6–12], which have led to a certain amount of progress.