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On the Classical Limit of Berry's Phase Integrable Systems

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Abstract. Berry's Phase is given by integration of a characteristic two form. We consider integrable systems defined by Weyl quantized classical Hamiltonians. It is shown that the limit of $\hbar/i$ times this two form is the curvature of the classical connection whose holonomy is the Hannay angles. A result of this type was derived by Berry [B2].

Introduction

Consider a quantum system whose dynamics is determined by a family of selfadjoint Hamiltonians on a Hilbert space depending smoothly on several parameters. Consider furthermore a region of energies such that the corresponding spectral subspace – defined by a projection P – also varies in a smooth manner.

Thus a vector bundle over the manifold of parameters is defined. As it is embedded in the trivial Hilbert bundle Pd is a natural connection on it – d denotes differentiation with respect to the parameters. It is well known [K], [A-S-Y] that as the parameters are driven adiabatically slowly in time the dynamical evolution starting in the spectral subspace is essentially the parallel – adiabatic – movement induced by this connection.

Curvature and holonomy effects have gained considerable interest in chemistry and physics. This was initiated by the work of Mead-Truhlar [M-T], Berry [B] and Simon [S] who first recognized the geometrical meaning.

We are interested in the quantum two form tr $PdPdP$ which represents the first Chern class of the bundle. Integrated over the interior of a contractible loop it gives Berry's Phase in the case of a line bundle.

Let the operators be given by quantization of classical Hamiltonians. All objects then depend on a semiclassical parameter $\hbar$. Consider a sequence of eigenvalues approaching a given classical energy as $\hbar$ goes to zero and the corresponding eigenprojections P.