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Positivity of Wightman Functionals and the Existence of Local Nets

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Abstract. The paper is concerned with the existence of a local net of von Neumann algebras associated with a given Wightman field. For fields satisfying a generalized H-bound the existence of such a net is shown to be equivalent to a certain positivity property of the Wightman distributions.

1. Introduction

The connection of Wightman quantum field theory [25, 20] with the theory of local nets of C^*- or von Neumann algebras [19, 2, 3] has been the subject of a number of investigations during the past 25 years, cf. e.g. [11, 17, 15, 4, 5, 13, 16, 23, 28, 14, 26, 1, 29, 12, 30]. The present note is concerned with one aspect of this problem, viz. to formulate conditions on the Wightman distributions that ensure the existence of a corresponding local net of von Neumann algebras on the Hilbert space of the field.

Before we proceed it is necessary to make precise what it means to associate a Wightman field to a local net of von Neumann algebras. For notational simplicity we shall here only deal with the case of a single, hermitian, scalar field Φ. By a local net of von Neumann algebras we mean an assignment $R \mapsto \mathscr{A}(R)$ of regions R in Minkowski space $\mathbb{R}^d$ to von Neumann algebras $\mathscr{A}(R)$ on the Hilbert space of the field such that the usual conditions of isotony, locality and covariance are fulfilled [2, 3, 19]. It is convenient and for most purposes sufficient to restrict the choice of regions R to the following types: Closed double cones K, wedge domains W (bounded by two light-like hyperplanes), and causal complements, K^c and W^c of such domains.

A field can be associated to a net in different ways, cf. [14]. We shall use the following simple notion:

1.1. Definition. A Wightman field Φ is associated to a local net $\mathscr{A}$ of von Neumann algebras if each field operator $\Phi(f)$ has an extension to a closed operator, $\Phi(f)_e \subset \Phi(f^*)^*$, that is affiliated with the von Neumann algebra $\mathscr{A}(R)$ if the support of the test function f is contained in the interior of R.