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Zeta Functions and Transfer Operators for Piecewise Monotone Transformations*

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Abstract. Given a piecewise monotone transformation T of the interval and a piecewise continuous complex weight function g of bounded variation, we prove that the Ruelle zeta function $\zeta(z)$ of (T, g) extends meromorphically to $\{|z|<\theta^{-1}\}$ (where $\theta=\lim_{n\to\infty}\|g\circ T^{n-1}\cdot\dots\cdot g\circ T\cdot g\|_\infty^{1/n}$) and that z is a pole of ζ if and only if z^{-1} is an eigenvalue of the corresponding transfer operator $\mathscr{L}$. We do not assume that $\mathscr{L}$ leaves a reference measure invariant.

1. Introduction and Statement of Results

Suppose $T:[0,1]\to[0,1]$ is piecewise monotone, i.e., there is a finite partition $\mathscr{Z}$ of $[0,1]$ into intervals such that $T_{|Z}$ is strictly monotone and continuous for each $Z\in\mathscr{Z}$. For a function $f:[0,1]\to\mathbb{C}$, let

$$\operatorname{var}(f)=\sup\left\{\sum_{i=1}^{n}|f(a_i)-f(a_{i-1})|:n\geqq 1, 0\leqq a_0<\dots<a_n\leqq 1\right\},$$

$$\|f\|_{BV}=\operatorname{var}(f)+\sup(|f|),$$

and denote by $BV=\{f:[0,1]\to\mathbb{C}$ such that $\|f\|_{BV}<\infty\}$ the space of functions of bounded variation.

Given $g\in BV$, one can define the transfer operator

$$\mathscr{L}:BV\to BV,\quad \mathscr{L}f(x)=\sum_{y:T(y)=x}(f\cdot g)(y)=\sum_{Z\in\mathscr{Z}}(f\cdot g)\circ T_{|Z}^{-1}(x)$$

and the Ruelle zeta function

$$\zeta(z)=\exp\left(\sum_{n=1}^{\infty}\frac{z^n}{n}\sum_{x=T^n x}g_n(x)\right),$$

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