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The Non-Relativistic Coulomb Problem on a Cone

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Abstract. We study the non-relativistic Coulomb problem on a cone. The non-trivial topology of the cone breaks the symmetry associated with the conservation of the Lagrange–Laplace–Runge–Lenz vector. Classically this translates into a precession of the orbits, and quantum-mechanically into a splitting of the energy levels. For the scattering problem we find that classical multi-scattering is possible and that it gives rise to a wake structure; we also evaluate the full quantum wave function and from it recover the classical results.

1. Introduction

There has been considerable interest recently in the problem of scattering, both classical and quantum, by a cone [1–3], that is the behaviour of geodesics or solutions of the Klein–Gordon or Schrödinger equation in the background metric

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\phi^2, \tag{1.1}$$

where

$$-\pi\alpha \leqq \phi \leqq \pi\alpha \equiv \pi - \frac{\delta}{2}$$

and δ is the deficit angle of the cone. The purpose of the present paper is to extend this work to the case when a Coulomb potential of the form

$$V = -\frac{K}{r} \tag{1.2}$$

is present. Such a term may arise in a number of contexts including:

1. The induced electrostatic *repulsion* experienced by a particle with electric charge

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