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Fock Representations and BRST Cohomology in $SL(2)$ Current Algebra

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Abstract. We investigate the structure of the Fock modules over $A_1^{(1)}$ introduced by Wakimoto. We show that irreducible highest weight modules arise as degree zero cohomology groups in a BRST-like complex of Fock modules. Chiral primary fields are constructed as BRST invariant operators acting on Fock modules. As a result, we obtain a free field representation of correlation functions of the $SU(2)$ WZW model on the plane and on the torus. We also consider representations of fractional level arising in Polyakov's $2D$ quantum gravity. Finally, we give a geometrical, Borel–Weil-like interpretation of the Wakimoto construction.

1. Introduction

Kac–Moody Lie algebras play a central role in two-dimensional conformal field theory [1]: it appears that most known examples of conformal field theory models can be understood in terms of WZW models [2, 3] (by a coset construction [4]) whose symmetry algebra is a Kac–Moody algebra. It is therefore important to understand the structure of their representations, and of chiral primary fields, which are tensor operators for these algebras. In this paper we focus on the algebra $A_1^{(1)}$, the central extension of the Lie algebra of loops in $sl(2, \mathbf{C})$.

A very powerful method for explicitly constructing representations, chiral primary fields and their correlation functions in conformal field theory is the Feigin–Fuchs construction [5] in terms of free fields, first considered in the case of the Virasoro algebra. It was recently shown [6] that this construction relies on a hidden BRST-like symmetry, which realizes the space of physical states as a subquotient of the free field Fock space. This observation led to an integral expression for correlation functions of minimal models on the torus.

Here we extend these results to the $A_1^{(1)}$ Lie algebra. The free field representation spaces (Fock modules) for this algebra were introduced by Wakimoto [7]. They are labeled by a spin J and a level K, which can be arbitrary complex numbers. The corresponding Feigin–Fuchs-like construction was proposed by Zamolodchikov [8]. The case of interest to us is the case where $K+2=p/p'$ is a