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The Universal R-Matrix for $U_q sl(3)$ and Beyond!

Nigel Burroughs*

Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge, England CB3 9EW

Abstract. The R-matrices for the quantised Lie algebras A_n are constructed through the quantum double procedure given by Drinfel'd [6]. The case of $U_q sl(3)$ is thoroughly analysed initially to demonstrate the more subtle points of the calculation. The ease of the calculation for A_n is very dependent on a choice of generators for the Borel subalgebra $U_q b_+$ and its dual, and a certain ordering imposed on these generators which is related to the length of a certain word in the Weyl group.

Introduction

To every Lia algebra and Kac Moody algebra g there exists a unique Hopf algebra A; a one parameter deformation of the universal enveloping algebra of g. This is the quantisation of the algebra g, and was defined by Drinfel'd [6] and Jimbo [11]. In the terminology of [6], these Hopf algebras turn out to be (pseudo) quasi-triangular Hopf algebras, which means that there exists an element $R \in A \otimes A$, called the universal R-matrix, that satisfies certain properties. The recent interest in quantum groups and the associated quantised algebra appears to be based on two of these properties: the R-matrix is the quantisation of the classical r-matrix [2] associated with g, and R satisfies the quantum Yang Baxter equation. The former property is important in attempts to quantise Toda field theories and related systems, since the classical r-matrix defines the Poisson structure of the monodromy matrix [8]:

$$\{T \overset{\otimes}{,} T\} = [r, T \otimes T], \tag{1}$$

where any variable dependence of the monodromy matrix T and classical r-matrix r (in some representation) has been suppressed. Quantisation is then achieved by interpreting T as a matrix of operators that satisfies an appropriate quantum level

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