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Decay Estimates for Schrödinger Equations

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Abstract. We prove global existence and optimal decay estimates for classical solutions with small initial data for nonlinear nonlocal Schrödinger equations. The Laplacian in the Schrödinger equation can be replaced by an operator corresponding to a non-degenerate quadratic form of arbitrary signature. In particular, the Davey–Stewartson system is included in the the class of equations we discuss.

Introduction

Nonlinear Schrödinger systems arise naturally as envelope equations in the study of water waves ([N, D–S, Z–K]). Their form is

$$i\frac{\partial}{\partial t}u + L_1 u = a|u|^2 u + vu, \tag{0.1}$$

$$L_2 v = L_3(|u|^2), \tag{0.2}$$

where a is real and L_1, L_2, L_3 are quadratic differential operators

$$L_l = g_l^{jk}\frac{\partial^2}{\partial x^j \partial x^k} \tag{0.3}$$

for $l = 1, 2, 3$. The constant real n by n matrices (g_l^{jk}) are invertible but otherwise general. In this paper we assume L_2 to be elliptic; in this case one can solve for v in (0.2) and write the system (0.1), (0.2) as a single equation:

$$i\frac{\partial}{\partial t}u + L_1 u = L(|u|^2)u, \tag{0.4}$$

where $L = aI + (L_2)^{-1}L_3$ is a linear operator which commutes with translations, is real and it is bounded as an operator from $L^p(\mathbb{R}^n)$ to itself for $1 < p < \infty$. The

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