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Classical and Thermodynamic Limits for Generalised Quantum Spin Systems

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Abstract. We prove that the rescaled upper and lower symbols for arbitrary generalised quantum spin systems converge in the classical limit. For a large class of models this enables us to derive the asymptotics of quantum free energies in the classical and in the thermodynamic limit.

1. Introduction

Several authors have studied the classical limit of quantum partition functions based on compact Lie groups. Particular cases were studied: first by Lieb [1] for $SU(2)$, and then by Fuller and Lenard [2] for $O(n)$, and Gilmore [3] for $SU(n)$. A unified treatment for general compact semi-simple Lie groups was given by Simon [4]. However, Simon's techniques were only successful for quantum systems built upon fully symmetric group representations and, as far as we are aware, this gap in the theory has not been filled. The contribution of this paper is to obtain classical and thermodynamic limits for quantum systems built upon arbitrary representations of compact semi-simple Lie groups. Furthermore, we are able to treat arbitrary polynomial Hamiltonians, rather than just the multiaffine Hamiltonians described in [4]. Our results rest on the proof of general limit theorems for the upper and lower symbols of polynomial Hamiltonians: we show that in all cases they coalesce in the classical limit. By a well-known procedure, this allows the computation of quantum-free energies in the classical limit. Furthermore, for a general class of mean-field models we are able to calculate the free-energies in the rather more interesting case of the thermodynamic limit.

In this introduction we will describe our framework and state our main technical result. We shall then summarize the contents of the paper.

Let G be a compact semi-simple Lie group with Lie algebra $\mathbf{g}, H$ a maximal abelian subgroup of G with Lie sub-algebra $\mathbf{h}$. Recall [5] that there exists a set of elements $\{\lambda_i : i \in E = \{1, 2, \ldots, \mathrm{rank}(G)\}\}$ of $\mathbf{h}^* = (i\mathbf{h})'$, the real dual of $i\mathbf{h}$, such that the

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