

Representations of the Heisenberg Algebra on a Riemann Surface^{*}

Arthur Jaffe, Slawomir Klimek, and Andrzej Lesniewski

Harvard University, Cambridge, MA 02138, USA

Abstract. We formulate reflection positivity for meromorphic functions and for 1-forms on a Riemann surface. This construction yields representations of the Heisenberg algebra on a Riemann surface.

I. Introduction

Conformal field theories on a Riemann surface with genus ≥ 1 have interesting features which are absent in the case of genus zero. One example of such a phenomenon is the nonuniqueness of basic representations of global chiral algebras. The simplest example of such an algebra is the Heisenberg algebra. Study of this algebra yields the theory of a free chiral field on a Riemann surface.

Krichever and Novikov understood the fact that the ordinary Heisenberg algebra, suitable for surfaces of genus zero, has to be extended [KN]. We study the resulting algebra in Sects. II and III; ultimately in Sect. V we describe global representations for such an algebra.

We restrict attention to Riemann surfaces S which are Schottky doubles of an open Riemann surface T . On such a Riemann surface there is a natural notion of Osterwalder-Schrader positivity. In other words, there is an antiholomorphic involution ϑ which we can interpret as time reflection. We study a space \mathfrak{A} of meromorphic functions on S . We use Θ , the lift of ϑ to \mathfrak{A} , and a symplectic form (\cdot, \cdot) on \mathfrak{A} ; we then define an inner product $\langle \cdot, \cdot \rangle = (\Theta \cdot, \cdot)$ on a subalgebra \mathfrak{A}_+ of \mathfrak{A} . The existence of an inner product allows us to construct representations of \mathfrak{A} on a Fock-type Hilbert space. Our method uses certain ideas in [DVV].

Riemann surfaces which are Schottky doubles exist for any genus. In conformal field theory one has developed methods to compute arbitrary correlation functions once the operator formalism is known for certain simple surfaces. Here we do not use such sewing procedures, but rather give a direct, global construction.

^{*} Supported in part by the Department of Energy under Grant DE-FG02-88ER25065