Commun. Math. Phys. 126, 367–378 (1989)

Communications in
Mathematical
Physics

Continuum Analogues of Contragredient Lie Algebras

(Lie Algebras with a Cartan Operator and Nonlinear Dynamical Systems)

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Abstract. We present an axiomatic formulation of a new class of infinite-dimensional Lie algebras – the generalizations of Z-graded Lie algebras with, generally speaking, an infinite-dimensional Cartan subalgebra and a contiguous set of roots. We call such algebras "continuum Lie algebras." The simple Lie algebras of constant growth are encapsulated in our formulation. We pay particular attention to the case when the local algebra is parametrized by a commutative algebra while the Cartan operator (the generalization of the Cartan matrix) is a linear operator. Special examples of these algebras are the Kac-Moody algebras, algebras of Poisson brackets, algebras of vector fields on a manifold, current algebras, and algebras with differential or integro-differential Cartan operator. The nonlinear dynamical systems associated with the continuum contragredient Lie algebras are also considered.

Introduction

In this paper we present an axiomatic formulation and give the principal examples of continuum generalizations of Z-graded algebras with generally speaking, an infinite-dimensional Cartan subalgebra. Our construction includes the simple Lie algebras of constant growth. Very special cases of these algebras have been discussed previously[1]. However, their (more or less) precise definition, albeit rather imperfect, was given in [1]. There, the discovery of the continuum algebras (called "continual Lie algebras" there) was stimulated by an investigation of nonlinear dynamical systems. On the other hand, already in the 60's and 70's, associative

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[1] As an example note paper [5] in which dynamical systems are generated by the associative algebras of integral operators. These algebras are defined in the space of measurable functions on an arbitrary set M with a measure preserving invertible transformation $M: M \to M$. They form, in particular, some subclass of the algebras considered in Example 6 in Sect. 2