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First Order Phase Transitions in Unbounded Spin Systems *I*: Construction of the Phase Diagram

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Abstract. The phase diagram and the corresponding infinite volume Gibbs states are constructed for a large class of continuous, unbounded spin models. Our construction relies on a partition of unity mapping our system onto an interacting contour system, a generalisation of Zahradnik's approach to Piragov Sinai theory to interacting contour systems, and a suitable mean field expansion around the minimas of the Hamiltonian.

Introduction

In this and a forthcoming paper, [1], we study continuous spin models with a single spin potential, V, whose minima are deep and widely separated. While such models have been extensively discussed in the literature if the minima of V are related by a symmetry, [2–5], results concerning the general case without any symmetry were only obtained in recent years.

The first work in this direction is that of Imbrie, [6], who treated two dimensional Euclidean field theories in which the potential, V, is a polynomial in the fields. His idea was to use the cluster expansion of [2] to obtain a hard core interacting contour system and to analyse this system using the techniques of Pirogov and Sinai, [7–9]. This turned out to be technically rather complicated because the resulting contour activities were not positive. Imbrie solved this problem by using a relatively involved resummation technique[1].

An alternative approach would be that of Bricmont et al., [12], who mapped certain lattice (and continuum) gases with three or more particle species to an interacting contour model (with positive activities) and then studied this model using the methods of [7–9]. Their method has been extended to bounded continuous spin systems on the lattice, but it is not clear whether their techniques extend to the unbounded case as well.

[1] Recently, Borgs and Imbrie realised, [10], that this resummation can be avoided using the techniques of [11] rather than [7–9] since that approach may be generalised to contour models with complex activities. See [10] for details