

Index of Subfactors and Statistics of Quantum Fields. I

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Abstract. We identify the statistical dimension of a superselection sector in a local quantum field theory with the square root of the index of a localized endomorphism of the quasi-local C^* -algebra that represents the sector. As a consequence in a two-dimensional theory the possible values of the statistical dimension below 2 are restricted to a given discrete set.

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1. Introduction

In this paper we shall exhibit a natural connection between the statistics of local quantum fields and the index theory of subfactors. In particular the statistical dimension of a superselection sector [7] will appear as the square root of the index of an associated inclusion of von Neumann algebras [19]. The restriction on the possible values of the index [19] then imposes a corresponding restriction on the possible values of the statistical dimension. In particular for a two space-time dimensional quantum field theory the range of the statistical dimension consists of a discrete series and possibly of a continuous part.

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