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The Chern-Simons Theory and Knot Polynomials

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Abstract. The Chern-Simons gauge theory is studied using a functional integral quantization. This leads to a differential equation for expectations of Wilson lines. The solution of this differential equation is shown to be simply related to the two-variable Jones polynomial of the corresponding link, in the case where the gauge group is $SU(N)$. A similar equation has been used before to get the Jones polynomial from a braid representation of the link. The main novelty of our approach is that we get the Jones polynomial from a plat representation of the link.

1. Introduction

The purpose of this paper is to explore the connection between the non-abelian Chern-Simons gauge theory and knot polynomials. The motivation for this work comes, in part, from Witten's beautiful paper [1] in which he shows that the expectation of a collection of Wilson loops in the Chern-Simons theory is related to the Jones polynomial of the corresponding link.

Our approach is different from that in [1]. We start from the classical action of Chern-Simons theory in Minkowski space and impose a particular gauge condition which we call *light-cone gauge.* In this gauge the action is quadratic in the gauge potential. The Feynman path integral then formally yields the propagator for the gauge potential in light-cone gauge, up to an overall normalization constant, denoted λ. Next, we complexify space-time and analytically continue the propagator from the Minkowski space to the Euclidean region. In the Euclidean region we may then derive simple differential equations for the expectations of Wilson lines in light-cone gauge with coefficients depending on λ. These equations are similar to equations studied previously in connection with representations of the braid groups [2, 3] and the Wess-Zumino-Witten models of two-dimensional conformal field theory, [4]. If we now require that the solutions of our differential equations for expectations of Wilson lines are compatible with unitarity (reflection positivity) of the theory then the parameter λ is constrained to