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Classification of Lattices with Z_m Symmetry

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Abstract. We consider the n-dimensional Euclidean lattices with Z_m symmetries. It is shown that such lattices can be considered as ideals of some cyclotomic fields. Therefore we can translate problems about the above lattices into those about number theory. For all n ($n \leqq 22$), we have obtained the classification of such lattices.

1. Introduction

In recent years superstring theory has been actively investigated. Since superstring theory has the critical dimension 10, we must consider compactification problems. Recently, many compactification schemes have been studied intensively [1–4]. Especially, since orbifold models [1] are phenomenologically interesting and easy to handle, many orbifold models have been constructed [5–7]. Therefore, not only phenomenological considerations, but also systematic classifications of orbifold models are needed [7].

An orbifold is a torus divided by its automorphisms, and the torus is an Euclidean space divided by some lattice. Therefore, in order to classify orbifold models, we have to classify lattices, at first. So in this paper, we consider lattices which have Z_m symmetries.

This paper is organized as follows. In Sect. 2, we clarify the problem by considering it form the viewpoint of eigenvalues of automorphism transformations. In Sect. 3, we study cases in which lattices have special symmetries. We show that in these cases, lattices can be considered as ideals of some cyclotomic field. Furthermore, we classify such special lattices. In Sect. 4, we classify general lattices by making use of the results which are obtained in Sect. 2 and 3. Section 5 is devoted to a conclusion.

Throughout this paper, we assume some knowledge of number theory. About number theory, we refer to [8–11]. Especially about cyclotomic fields, we refer to [11].