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Large-Mass and High-Temperature Behaviour in Perturbative Quantum Field Theory

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Abstract. Bounds for large-mass behaviour in renormalized perturbation expansions at zero temperature, which were previously obtained by Manoukian and Caswell-Kennedy in momentum space, are rederived in the parametric representation. A very simple unified proof of the BPHZ theorem and the decoupling theorem is also given. A new technique for asymptotic analysis, based on a generalized Kontorovich-Lebedev integral transform, is introduced. This method is applied to find the leading high-temperature behaviour of perturbative field theories in the imaginary-time formalism. We prove that diagrams containing nonstatic modes, which at high temperature behave like particles with a large mass, are suppressed relative to purely static diagrams. This rigorously proves a limited form of dimensional reduction at infinite temperature.

1. Introduction

In the early eighties it has been suggested that at very high temperatures field theories in the imaginary-time (Matsubara) formalism would undergo a form of dimensional reduction [5, 14, 16]. The presence of a nonzero temperature T can be incorporated in quantum field theory by compactifying the Euclidean time axis to a circle with radius $\beta = T^{-1}$ [18], and this obviously motivates the idea of a dimensional reduction from d to $d-1$ dimensions.

In perturbation theory in momentum space the temperature enters in the guise of a mass $m = 2\pi n T$, $n \in Z$, which is present in each (bosonic) propagator [18]. The analogy with the decoupling theorem for heavy particles [4, 2, 20, 9] then suggests that at high temperatures the nonzero modes ($n \neq 0$) decouple at low momenta, leaving an effective three-dimensional theory, consisting of the zero modes only, behind. In contradistinction to the ordinary decoupling theorem, it is here supposed that an infinite tower of massive particles decouples.

In order to prove that such a dimensional reduction mechanism indeed applies, one ought to state a renormalization scheme in which the nonstatic modes decouple